Edexcel GCSE Mathematics (Higher Tier)

Quadratic Equations

Fully Worked Solutions: every line explained, mark-scheme annotated

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40 questions | Progressive difficulty (Easy → Hard) | Algebra (1MA1)

Created by Aaditya Kalwani

Key methods:

$$\text{quadratic formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad \text{discriminant: } b^2 - 4ac$$

Which method? If it factorises with whole numbers, factorise. If not, complete the square (for exact form or turning points) or use the quadratic formula. The discriminant $b^2 - 4ac$ tells you the number of real roots: positive gives two, zero gives one (repeated), negative gives none.

Questions

Easy

Q1) Solve $x^2 + 5x + 6 = 0$. **[2 marks]**

Q2) Solve $x^2 - 7x + 12 = 0$. **[2 marks]**

Q3) Solve $x^2 - x - 12 = 0$. **[2 marks]**

Q4) Solve $x^2 + 2x - 15 = 0$. **[2 marks]**

Q5) Solve $x^2 - 9 = 0$. **[2 marks]**

Q6) Solve $x^2 - 6x = 0$. **[2 marks]**

Q7) Factorise $x^2 + 8x + 15$. **[2 marks]**

Q8) Factorise $x^2 - 25$. **[2 marks]**

Q9) Solve $x^2 = 16$. **[2 marks]**

Q10) Solve $x^2 + 4x + 4 = 0$. **[2 marks]**

Q11) Solve $x^2 - 10x + 25 = 0$. **[2 marks]**

Q12) Factorise $x^2 - 3x - 10$. **[2 marks]**

Q13) Solve $(x - 2)(x + 5) = 0$. **[1 mark]**

Q14) Solve $x^2 + 3x - 10 = 0$. **[2 marks]**

Standard

Q15) Solve $2x^2 + 7x + 3 = 0$. **[3 marks]**

Q16) Solve $3x^2 - 5x - 2 = 0$. **[3 marks]**

Q17) Solve $6x^2 + x - 2 = 0$. **[3 marks]**

Q18) Solve $x^2 + 4x + 1 = 0$, giving your answers in exact (surd) form. **[3 marks]**

Q19) Solve $x^2 - 6x + 4 = 0$, giving your answers in exact form. **[3 marks]**

Q20) Solve $2x^2 - 4x - 3 = 0$, giving your answers in exact form. **[3 marks]**

Q21) Write $x^2 + 6x + 5$ in the form $(x + a)^2 + b$. **[2 marks]**

Q22) Write $x^2 - 4x + 9$ in the form $(x + a)^2 + b$. **[2 marks]**

Q23) By completing the square, solve $x^2 + 8x + 10 = 0$, giving exact answers. **[4 marks]**

Q24) Solve $5x^2 - 3x - 2 = 0$. **[3 marks]**

Q25) Solve $x^2 - 2x - 5 = 0$, giving your answers in exact form. **[3 marks]**

Q26) Write $2x^2 + 8x + 3$ in the form $a(x + b)^2 + c$. **[3 marks]**

Q27) Solve $4x^2 - 9 = 0$. **[2 marks]**

Q28) Solve $3x^2 = 12x$. **[2 marks]**

Q29) The product of two consecutive positive integers is 56. By forming and solving a quadratic equation, find the two integers. **[4 marks]**

Challenging

Q30) Find the value of the discriminant of $2x^2+3x+5 = 0$ and state how many real roots the equation has. **[3 marks]**

Q31) The equation $x^2+kx+9 = 0$ has equal roots. Find the possible values of k. **[3 marks]**

Q32) Find the range of values of k for which $x^2 + kx + 4 = 0$ has two distinct real roots. **[4 marks]**

Q33) Solve simultaneously $y = x^2 - 3x + 2$ and $y = x - 1$. **[5 marks]**

Q34) Solve simultaneously $y = x + 4$ and $x^2 + y^2 = 16$. **[5 marks]**

Q35) By completing the square, find the coordinates of the turning point of $y = x^2 - 6x + 11$. **[4 marks]**

Q36) By completing the square, find the coordinates of the turning point of $y = 2x^2 + 4x - 1$. **[4 marks]**

Q37) Solve the inequality $x^2 - x - 6 > 0$. **[4 marks]**

Q38) Solve the inequality $x^2 - 5x + 4 \leq 0$. **[4 marks]**

Q39) A rectangle has length $(x + 3)$ cm and width x cm. Its area is 40 cm^2. Find the value of x. **[4 marks]**

Q40) By completing the square, prove that $x^2 - 8x + 20$ is positive for all real values of x. **[4 marks]**

Worked Solutions

Q1) Answer: $x = -2$ **or** $x = -3$

Worked Solution:

Step 1: Factorise

Find two numbers that multiply to $+6$ and add to $+5$: these are 2 and 3.

$$x^2 + 5x + 6 = (x + 2)(x + 3) = 0 \qquad \leftarrow \textit{factorise} \;\; \textbf{[M1]}$$

Step 2: Set each bracket to zero

$$x + 2 = 0 \text{ or } x + 3 = 0 \implies x = -2 \text{ or } x = -3 \qquad \leftarrow \textit{solve} \;\; \textbf{[A1]}$$

$$x = -2 \text{ or } x = -3$$

Q2) Answer: $x = 3$ **or** $x = 4$

Worked Solution:

Step 1: Factorise

Two numbers multiplying to $+12$ and adding to -7: these are -3 and -4.

$$x^2 - 7x + 12 = (x - 3)(x - 4) = 0 \qquad \leftarrow \textit{factorise} \;\; \textbf{[M1]}$$

$$x = 3 \text{ or } x = 4 \qquad \leftarrow \textit{solve} \;\; \textbf{[A1]}$$

$$x = 3 \text{ or } x = 4$$

Q3) Answer: $x = 4$ **or** $x = -3$

Worked Solution:

Step 1: Factorise

Product -12, sum -1: the numbers are -4 and $+3$.

$$x^2 - x - 12 = (x - 4)(x + 3) = 0 \qquad \leftarrow \textit{factorise} \;\; \textbf{[M1]}$$

$$x = 4 \text{ or } x = -3 \qquad \leftarrow \textit{solve} \;\; \textbf{[A1]}$$

$$x = 4 \text{ or } x = -3$$

Q4) Answer: $x = -5$ **or** $x = 3$

Worked Solution:

Step 1: Factorise

Product -15, sum $+2$: the numbers are $+5$ and -3.

$$x^2 + 2x - 15 = (x + 5)(x - 3) = 0 \quad \leftarrow \textit{factorise} \ \textit{[M1]}$$
$$x = -5 \text{ or } x = 3 \quad \leftarrow \textit{solve} \ \textit{[A1]}$$

$$x = -5 \text{ or } x = 3$$

Q5) Answer: $x = \pm 3$

Worked Solution:

Step 1: Recognise the difference of two squares

$$x^2 - 9 = (x - 3)(x + 3) = 0 \quad \leftarrow \textit{difference of squares} \ \textit{[M1]}$$
$$x = 3 \text{ or } x = -3 \quad \leftarrow \textit{solve} \ \textit{[A1]}$$

$$x = \pm 3$$

Q6) Answer: $x = 0$ **or** $x = 6$

Worked Solution:

Step 1: Take out the common factor x

Do not divide by x, that would lose the root $x = 0$.

$$x^2 - 6x = x(x - 6) = 0 \quad \leftarrow \textit{common factor} \ \textit{[M1]}$$
$$x = 0 \text{ or } x = 6 \quad \leftarrow \textit{solve} \ \textit{[A1]}$$

$$x = 0 \text{ or } x = 6$$

Q7) Answer: $(x + 3)(x + 5)$

Worked Solution:

Step 1: Two numbers with product 15 and sum 8

$$x^2 + 8x + 15 = (x+3)(x+5) \qquad \leftarrow 3 \times 5 = 15,\ 3 + 5 = 8 \quad \textit{[M1 A1]}$$

$$x^2 + 8x + 15 = (x+3)(x+5)$$

Q8) Answer: $(x-5)(x+5)$

Worked Solution:

Step 1: Difference of two squares
$x^2 - 25 = x^2 - 5^2$.

$$x^2 - 25 = (x-5)(x+5) \qquad \leftarrow a^2 - b^2 = (a-b)(a+b) \quad \textit{[M1 A1]}$$

$$x^2 - 25 = (x-5)(x+5)$$

Q9) Answer: $x = \pm 4$

Worked Solution:

Step 1: Take the square root of both sides
Remember to include both the positive and negative root.

$$x^2 = 16 \implies x = \pm\sqrt{16} = \pm 4 \qquad \leftarrow \pm \textit{ both roots} \quad \textit{[M1 A1]}$$

$$x = \pm 4$$

Q10) Answer: $x = -2$ **(repeated)**

Worked Solution:

Step 1: Recognise a perfect square

$$x^2 + 4x + 4 = (x+2)^2 = 0 \qquad \leftarrow \textit{perfect square} \quad \textit{[M1]}$$
$$x = -2 \qquad \leftarrow \textit{repeated root} \quad \textit{[A1]}$$

$$x = -2 \text{ (repeated root)}$$

Q11) Answer: $x = 5$ **(repeated)**

Worked Solution:

Step 1: Recognise a perfect square

$$x^2 - 10x + 25 = (x-5)^2 = 0 \qquad \leftarrow \textit{perfect square } \textbf{[M1]}$$
$$x = 5 \qquad \leftarrow \textit{repeated root } \textbf{[A1]}$$

$x = 5$ (repeated root)

Q12) Answer: $(x-5)(x+2)$

Worked Solution:

Step 1: Product -10, sum -3

$$x^2 - 3x - 10 = (x-5)(x+2) \qquad \leftarrow -5 \times 2 = -10,\ -5 + 2 = -3 \ \textbf{[M1 A1]}$$

$x^2 - 3x - 10 = (x-5)(x+2)$

Q13) Answer: $x = 2$ **or** $x = -5$

Worked Solution:

Step 1: A product is zero when a bracket is zero

$$(x-2)(x+5) = 0 \implies x = 2 \text{ or } x = -5 \qquad \leftarrow \textit{null factor law } \textbf{[B1]}$$

$x = 2$ or $x = -5$

Q14) Answer: $x = -5$ **or** $x = 2$

Worked Solution:

Step 1: Product -10, sum $+3$

$$x^2 + 3x - 10 = (x+5)(x-2) = 0 \qquad \leftarrow \textit{factorise } \textbf{[M1]}$$
$$x = -5 \text{ or } x = 2 \qquad \leftarrow \textit{solve } \textbf{[A1]}$$

$$x = -5 \text{ or } x = 2$$

Q15) Answer: $x = -\frac{1}{2}$ **or** $x = -3$

Worked Solution:

Step 1: Factorise with $a \neq 1$

Split the middle term: two numbers with product $ac = 2 \times 3 = 6$ and sum 7 are 6 and 1.

$$\begin{aligned} 2x^2 + 7x + 3 &= 2x^2 + 6x + x + 3 && \leftarrow \textit{split middle term} \quad \textbf{[M1]} \\ &= 2x(x+3) + 1(x+3) && \leftarrow \textit{factor in pairs} \\ &= (2x+1)(x+3) = 0 && \leftarrow \textit{factorise} \quad \textbf{[A1]} \end{aligned}$$

Step 2: Solve each bracket

$$x = -\tfrac{1}{2} \text{ or } x = -3 \qquad \leftarrow \textit{solve} \quad \textbf{[A1]}$$

$$x = -\tfrac{1}{2} \text{ or } x = -3$$

Q16) Answer: $x = -\frac{1}{3}$ **or** $x = 2$

Worked Solution:

Step 1: Split the middle term

Product $ac = 3 \times (-2) = -6$, sum -5: the numbers are -6 and $+1$.

$$\begin{aligned} 3x^2 - 5x - 2 &= 3x^2 - 6x + x - 2 && \leftarrow \textit{split middle term} \quad \textbf{[M1]} \\ &= 3x(x-2) + 1(x-2) = (3x+1)(x-2) = 0 && \leftarrow \textit{factorise} \quad \textbf{[A1]} \\ x &= -\tfrac{1}{3} \text{ or } x = 2 && \leftarrow \textit{solve} \quad \textbf{[A1]} \end{aligned}$$

$$x = -\tfrac{1}{3} \text{ or } x = 2$$

Q17) Answer: $x = -\frac{2}{3}$ **or** $x = \frac{1}{2}$

Worked Solution:

Step 1: Split the middle term

Product $ac = 6 \times (-2) = -12$, sum $+1$: the numbers are $+4$ and -3.

$$6x^2 + x - 2 = 6x^2 + 4x - 3x - 2 \quad \leftarrow \textit{split middle term } \textbf{[M1]}$$
$$= 2x(3x+2) - 1(3x+2) = (3x+2)(2x-1) = 0 \quad \leftarrow \textit{factorise } \textbf{[A1]}$$
$$x = -\tfrac{2}{3} \text{ or } x = \tfrac{1}{2} \quad \leftarrow \textit{solve } \textbf{[A1]}$$

$$x = -\tfrac{2}{3} \text{ or } x = \tfrac{1}{2}$$

Q18) Answer: $x = -2 \pm \sqrt{3}$

Worked Solution:

Step 1: Apply the quadratic formula

Here $a = 1$, $b = 4$, $c = 1$.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} \quad \leftarrow \textit{substitute into formula } \textbf{[M1]}$$
$$= \frac{-4 \pm \sqrt{12}}{2} \quad \leftarrow 16 - 4 = 12$$

Step 2: Simplify the surd

$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$, then cancel the 2.

$$x = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3} \quad \leftarrow \textit{simplify } \textbf{[A1 A1]}$$

$$x = -2 \pm \sqrt{3}$$

Q19) Answer: $x = 3 \pm \sqrt{5}$

Worked Solution:

Step 1: Quadratic formula with $a = 1$, $b = -6$, $c = 4$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(4)}}{2} = \frac{6 \pm \sqrt{20}}{2} \quad \leftarrow \textit{substitute } \textbf{[M1]}$$

Step 2: Simplify $\sqrt{20} = 2\sqrt{5}$

$$x = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5} \quad \leftarrow \textit{simplify } \textbf{[A1 A1]}$$

$$x = 3 \pm \sqrt{5}$$

Q20) Answer: $x = 1 \pm \frac{\sqrt{10}}{2}$

Worked Solution:

Step 1: Quadratic formula with $a = 2$, $b = -4$, $c = -3$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)} = \frac{4 \pm \sqrt{16 + 24}}{4} \quad \leftarrow \textit{substitute} \; \textbf{[M1]}$$

$$= \frac{4 \pm \sqrt{40}}{4} \quad \leftarrow 16 + 24 = 40$$

Step 2: Simplify $\sqrt{40} = 2\sqrt{10}$

$$x = \frac{4 \pm 2\sqrt{10}}{4} = 1 \pm \frac{\sqrt{10}}{2} \quad \leftarrow \textit{simplify} \; \textbf{[A1 A1]}$$

$$x = 1 \pm \frac{\sqrt{10}}{2}$$

Q21) Answer: $(x + 3)^2 - 4$

Worked Solution:

Step 1: Halve the coefficient of x
Half of 6 is 3, so the bracket is $(x + 3)^2$, which expands to $x^2 + 6x + 9$.

$$x^2 + 6x + 5 = (x + 3)^2 - 9 + 5 \quad \leftarrow \textit{complete the square} \; \textbf{[M1]}$$

$$= (x + 3)^2 - 4 \quad \leftarrow \textit{simplify} \; \textbf{[A1]}$$

$$x^2 + 6x + 5 = (x + 3)^2 - 4$$

Q22) Answer: $(x - 2)^2 + 5$

Worked Solution:

Step 1: Half of -4 is -2

$(x-2)^2 = x^2 - 4x + 4$, so subtract the 4 back off.

$$x^2 - 4x + 9 = (x-2)^2 - 4 + 9 \qquad \leftarrow \textit{complete the square } \textbf{[M1]}$$
$$= (x-2)^2 + 5 \qquad \leftarrow \textit{simplify } \textbf{[A1]}$$

$$x^2 - 4x + 9 = (x-2)^2 + 5$$

Q23) Answer: $x = -4 \pm \sqrt{6}$

Worked Solution:

Step 1: Complete the square

Half of 8 is 4, so $(x+4)^2 = x^2 + 8x + 16$.

$$x^2 + 8x + 10 = (x+4)^2 - 16 + 10 = (x+4)^2 - 6 \qquad \leftarrow \textit{complete the square } \textbf{[M1]}$$

Step 2: Solve $(x+4)^2 = 6$

$$(x+4)^2 - 6 = 0 \implies (x+4)^2 = 6 \qquad \leftarrow \textit{rearrange } \textbf{[M1]}$$
$$x + 4 = \pm\sqrt{6} \implies x = -4 \pm \sqrt{6} \qquad \leftarrow \textit{solve } \textbf{[A1 A1]}$$

$$x = -4 \pm \sqrt{6}$$

Q24) Answer: $x = -\frac{2}{5}$ **or** $x = 1$

Worked Solution:

Step 1: Split the middle term

Product $ac = 5 \times (-2) = -10$, sum -3: the numbers are -5 and $+2$.

$$5x^2 - 3x - 2 = 5x^2 - 5x + 2x - 2 \qquad \leftarrow \textit{split middle term } \textbf{[M1]}$$
$$= 5x(x-1) + 2(x-1) = (5x+2)(x-1) = 0 \qquad \leftarrow \textit{factorise } \textbf{[A1]}$$
$$x = -\tfrac{2}{5} \text{ or } x = 1 \qquad \leftarrow \textit{solve } \textbf{[A1]}$$

$$x = -\tfrac{2}{5} \text{ or } x = 1$$

Q25) Answer: $x = 1 \pm \sqrt{6}$

Worked Solution:

Step 1: Complete the square

Half of -2 is -1, so $(x-1)^2 = x^2 - 2x + 1$.

$$x^2 - 2x - 5 = (x-1)^2 - 1 - 5 = (x-1)^2 - 6$$ ← *complete the square* ***[M1]***

Step 2: Solve

$$(x-1)^2 = 6 \implies x = 1 \pm \sqrt{6}$$ ← *solve* ***[A1 A1]***

$$x = 1 \pm \sqrt{6}$$

Q26) Answer: $2(x+2)^2 - 5$

Worked Solution:

Step 1: Factor the 2 out of the first two terms

$$2x^2 + 8x + 3 = 2(x^2 + 4x) + 3$$ ← *factor out 2* ***[M1]***

Step 2: Complete the square inside the bracket

Half of 4 is 2; $(x+2)^2 = x^2 + 4x + 4$, so subtract 4 inside.

$$= 2\left[(x+2)^2 - 4\right] + 3$$ ← *complete square inside* ***[M1]***

$$= 2(x+2)^2 - 8 + 3 = 2(x+2)^2 - 5$$ ← *expand and simplify* ***[A1]***

$$2x^2 + 8x + 3 = 2(x+2)^2 - 5$$

Q27) Answer: $x = \pm\frac{3}{2}$

Worked Solution:

Step 1: Difference of two squares (or rearrange)

$$4x^2 - 9 = (2x-3)(2x+3) = 0$$ ← *difference of squares* ***[M1]***

$$x = \tfrac{3}{2} \text{ or } x = -\tfrac{3}{2}$$ ← *solve* ***[A1]***

$$x = \pm\tfrac{3}{2}$$

Q28) Answer: $x = 0$ **or** $x = 4$

Worked Solution:

Step 1: Bring everything to one side

Never divide by x; move all terms across so the root $x = 0$ is kept.

$$3x^2 - 12x = 0 \quad \leftarrow \textit{rearrange to } = 0 \ \textbf{[M1]}$$
$$3x(x-4) = 0 \quad \leftarrow \textit{common factor}$$
$$x = 0 \text{ or } x = 4 \quad \leftarrow \textit{solve} \ \textbf{[A1]}$$

$$x = 0 \text{ or } x = 4$$

Q29) Answer: 7 **and** 8

Worked Solution:

Step 1: Form the equation

Let the integers be n and $n+1$; their product is 56.

$$n(n+1) = 56 \quad \leftarrow \textit{form equation} \ \textbf{[M1]}$$
$$n^2 + n - 56 = 0 \quad \leftarrow \textit{expand to standard form} \ \textbf{[M1]}$$

Step 2: Solve and pick the valid root

Product -56, sum $+1$: the numbers are $+8$ and -7.

$$(n+8)(n-7) = 0 \implies n = -8 \text{ or } n = 7 \quad \leftarrow \textit{factorise} \ \textbf{[A1]}$$

Since the integers are positive, $n = 7$, giving 7 and 8.

The integers are 7 and 8 [A1]

Q30) Answer: -31**; no real roots**

Worked Solution:

The discriminant $b^2 - 4ac$ decides the number of real roots without solving: positive gives two, zero gives one, negative gives none.

Step 1: Substitute $a = 2$, $b = 3$, $c = 5$

$$b^2 - 4ac = 3^2 - 4(2)(5) \quad \leftarrow \textit{substitute} \ \textbf{[M1]}$$
$$= 9 - 40 = -31 \quad \leftarrow \textit{evaluate} \ \textbf{[A1]}$$

Step 2: Interpret the sign

The discriminant is negative, so there are no real roots.

$$b^2 - 4ac = -31 < 0 \implies \text{no real roots} \qquad \textbf{[A1]}$$

Q31) Answer: $k = \pm 6$

Worked Solution:

Step 1: Equal roots means discriminant $= 0$

With $a = 1$, $b = k$, $c = 9$.

$$b^2 - 4ac = 0 \implies k^2 - 4(1)(9) = 0 \qquad \leftarrow \textit{set discriminant to } 0 \ \textit{[M1]}$$

$$k^2 - 36 = 0 \qquad \leftarrow \textit{evaluate}$$

Step 2: Solve for k

$$k^2 = 36 \implies k = \pm 6 \qquad \leftarrow \textit{both roots} \ \textit{[A1 A1]}$$

$$k = \pm 6$$

Q32) Answer: $k < -4$ **or** $k > 4$

Worked Solution:

Two distinct real roots requires a strictly positive discriminant, which turns into a quadratic inequality in k.

Step 1: Set discriminant > 0 with $a = 1$, $b = k$, $c = 4$

$$b^2 - 4ac > 0 \implies k^2 - 16 > 0 \qquad \leftarrow \textit{discriminant} > 0 \ \textit{[M1]}$$

Step 2: Solve the inequality

$k^2 - 16 = (k-4)(k+4)$; the curve is above the axis outside the roots.

$$(k-4)(k+4) > 0 \implies k < -4 \text{ or } k > 4 \qquad \leftarrow \textit{outside the roots} \ \textit{[M1 A1]}$$

$$k < -4 \text{ or } k > 4$$

Q33) Answer: $(1, 0)$ **and** $(3, 2)$

Worked Solution:

Step 1: Set the two expressions for y equal

$$x^2 - 3x + 2 = x - 1 \qquad \leftarrow \textit{equate} \quad \textit{[M1]}$$
$$x^2 - 4x + 3 = 0 \qquad \leftarrow \textit{gather to one side} \quad \textit{[M1]}$$

Step 2: Solve for x

$$(x-1)(x-3) = 0 \implies x = 1 \text{ or } x = 3 \qquad \leftarrow \textit{factorise} \quad \textit{[A1]}$$

Step 3: Find y from the linear equation
Using $y = x - 1$: at $x = 1$, $y = 0$; at $x = 3$, $y = 2$.

$$(1, 0) \text{ and } (3, 2) \qquad \textbf{[A1 A1]}$$

Q34) Answer: $(0, 4)$ **and** $(-4, 0)$

Worked Solution:

Step 1: Substitute the line into the circle

$$x^2 + (x+4)^2 = 16 \qquad \leftarrow \textit{substitute } y = x + 4 \quad \textit{[M1]}$$
$$x^2 + x^2 + 8x + 16 = 16 \qquad \leftarrow \textit{expand the bracket} \quad \textit{[M1]}$$

Step 2: Simplify and solve

$$2x^2 + 8x = 0 \implies 2x(x+4) = 0 \qquad \leftarrow \textit{simplify and factor} \quad \textit{[A1]}$$
$$x = 0 \text{ or } x = -4 \qquad \leftarrow \textit{solve}$$

Step 3: Find $y = x + 4$ for each
At $x = 0$, $y = 4$; at $x = -4$, $y = 0$.

$$(0, 4) \text{ and } (-4, 0) \qquad \textbf{[A1 A1]}$$

Q35) Answer: $(3, 2)$**, a minimum**

Worked Solution:

Completing the square puts a quadratic in the form $(x-p)^2+q$, which reads off the turning point (p, q) directly.

Step 1: Complete the square
Half of -6 is -3, so $(x-3)^2 = x^2-6x+9$.

$$\begin{aligned} y = x^2-6x+11 &= (x-3)^2-9+11 && \leftarrow \textit{complete the square} \ \textbf{[M1]} \\ &= (x-3)^2+2 && \leftarrow \textit{simplify} \ \textbf{[A1]} \end{aligned}$$

Step 2: Read off the turning point
The squared term is smallest (zero) at $x=3$, giving $y=2$; the positive x^2 coefficient makes it a minimum.

$$\text{Turning point } (3, 2), \ \text{a minimum} \qquad \text{[A1]}$$

Q36) Answer: $(-1, -3)$**, a minimum**

Worked Solution:

Step 1: Factor the 2 out first

$$y = 2x^2+4x-1 = 2(x^2+2x)-1 \qquad \leftarrow \textit{factor out 2} \ \textbf{[M1]}$$

Step 2: Complete the square inside
Half of 2 is 1, so $(x+1)^2 = x^2+2x+1$; subtract 1 inside.

$$\begin{aligned} &= 2\left[(x+1)^2-1\right]-1 && \leftarrow \textit{complete square inside} \ \textbf{[M1]} \\ &= 2(x+1)^2-2-1 = 2(x+1)^2-3 && \leftarrow \textit{simplify} \ \textbf{[A1]} \end{aligned}$$

Step 3: Read off the turning point

$$\text{Turning point } (-1, -3), \ \text{a minimum} \qquad \text{[A1]}$$

Q37) Answer: $x < -2$ **or** $x > 3$

Worked Solution:

For a quadratic inequality, find the roots first, then decide which region satisfies it by sketching the upward parabola.

Step 1: Find the roots

$$x^2 - x - 6 = (x-3)(x+2) = 0 \implies x = 3 \text{ or } x = -2 \qquad \leftarrow \textit{factorise for roots} \quad \textit{[M1]}$$

Step 2: Choose the region for > 0

The parabola opens upward, so it is above the axis outside the roots.

$$(x-3)(x+2) > 0 \implies x < -2 \text{ or } x > 3 \qquad \leftarrow \textit{outside the roots} \quad \textit{[M1 A1]}$$

$$x < -2 \text{ or } x > 3$$

Q38) Answer: $1 \le x \le 4$

Worked Solution:

Step 1: Find the roots

$$x^2 - 5x + 4 = (x-1)(x-4) = 0 \implies x = 1 \text{ or } x = 4 \qquad \leftarrow \textit{factorise} \quad \textit{[M1]}$$

Step 2: Choose the region for ≤ 0

The upward parabola is below (or on) the axis between the roots, and the inequality is "$\le$", so the roots are included.

$$(x-1)(x-4) \le 0 \implies 1 \le x \le 4 \qquad \leftarrow \textit{between the roots} \quad \textit{[M1 A1]}$$

$$1 \le x \le 4$$

Q39) Answer: $x = 5$

Worked Solution:

Step 1: Form the area equation

$$x(x+3) = 40 \qquad \leftarrow \textit{area} = \textit{length} \times \textit{width} \quad \textit{[M1]}$$

$$x^2 + 3x - 40 = 0 \qquad \leftarrow \textit{expand to standard form} \quad \textit{[M1]}$$

Step 2: Solve and reject the invalid root

Product -40, sum $+3$: the numbers are $+8$ and -5.

$$(x+8)(x-5) = 0 \implies x = -8 \text{ or } x = 5 \qquad \leftarrow \textit{factorise} \quad \textit{[A1]}$$

A length cannot be negative, so $x = 5$.

$$x = 5 \text{ (reject } x = -8, \text{ a length)} \qquad \textbf{[A1]}$$

Q40) Answer: $(x-4)^2 + 4 > 0$ **for all** x **(shown)**

Worked Solution:

The idea: a completed square $(x-4)^2$ can never be negative, so adding a positive constant keeps the whole expression above zero.

Step 1: Complete the square
Half of -8 is -4, so $(x-4)^2 = x^2 - 8x + 16$.

$$x^2 - 8x + 20 = (x-4)^2 - 16 + 20 = (x-4)^2 + 4 \quad \leftarrow \textit{complete the square} \quad \textit{[M1 A1]}$$

Step 2: Argue the sign
A square is never negative, so $(x-4)^2 \geq 0$; adding 4 gives at least 4.

$$(x-4)^2 \geq 0 \implies (x-4)^2 + 4 \geq 4 > 0 \quad \leftarrow \textit{square is non-negative} \quad \textit{[M1]}$$

Therefore $x^2 - 8x + 20 > 0$ for all real x.

$$x^2 - 8x + 20 = (x-4)^2 + 4 \geq 4 > 0 \quad \blacksquare \qquad \textbf{[A1]}$$